

A Gallery of Graphs

Objectives

- To recognise the rules of a number of common algebraic relationships: $y = x^{-1}$, $y = x^{-2}$, $y = x^{1/2}$ and $x^2 + y^2 = 1$.
- To be able to sketch the graphs and simple transformations of these relations.

In the previous chapter quadratics written in ‘turning point form’ were considered. The technique of sketching their graphs through employing transformations of the graph of the basic quadratic $y = x^2$ was introduced.

In this chapter some other common algebraic relations will be considered. Methods similar to those developed in Chapter 4 will be used to sketch the graphs of these relations.

First, it is necessary to be able to recognise the basic rule of each type of relation and its corresponding graph.

In this chapter use of a CAS calculator is recommended to obtain the initial graphs.

5.1 Rectangular hyperbolas

Consider the rule $y = \frac{1}{x} = x^{-1}$, for $x \neq 0$.

We can construct a table of values for $y = \frac{1}{x}$ for values of x between -4 and 4 as follows.

x	-4	-3	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	3	4
y	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

We can plot these points and then connect the dots to produce a continuous curve.

A graph of this type is an example of a **rectangular hyperbola**.

It should be noted that when $x = 0$, y is undefined and there is no x -value that will produce the value $y = 0$.

As x approaches infinity in either direction, the value of y approaches zero. The following notation will be used to state this.

As $x \rightarrow \infty$, $y \rightarrow 0^+$. As $x \rightarrow -\infty$, $y \rightarrow 0^-$. These are read:

‘As x approaches infinity, y approaches 0 from the positive side’ and ‘as x approaches negative infinity, y approaches 0 from the negative side’.

As x approaches zero from either direction, the magnitude of y becomes very large. The following notation will be used to state this.

As $x \rightarrow 0^+$, $y \rightarrow \infty$ and as $x \rightarrow 0^-$, $y \rightarrow -\infty$. These are read:

‘As x approaches zero from the positive side, y approaches infinity’ and ‘as x approaches zero from the negative side, y approaches negative infinity.’

The graph approaches both the x -axis (the line $y = 0$) and the y -axis (the line $x = 0$) but does not cross either of these lines.

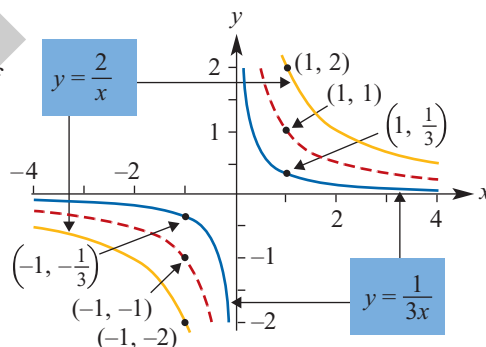
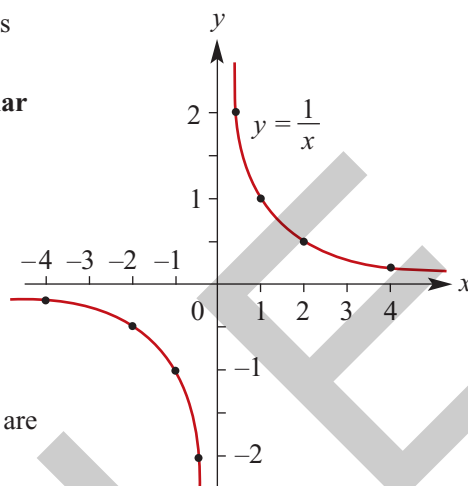
We refer to these lines as **asymptotes**.

Hence, for the graph of $y = \frac{1}{x}$, the equations of the asymptotes are $y = 0$ and $x = 0$.

In the diagram on the right, the graphs of $y = \frac{1}{x}$, $y = \frac{2}{x}$ and $y = \frac{1}{3x}$ are shown.

The asymptotes are the x -axis and the y -axis, and they have equations $y = 0$ and $x = 0$ respectively.

As can be seen from the diagram, the graphs of $y = \frac{2}{x}$ and $y = \frac{1}{3x}$ have the same ‘shape’ and asymptotes as $y = \frac{1}{x}$ but have been ‘stretched’ (or **dilated** from the x -axis). It is said that the **dilation** from the x -axis, which takes the graph of $y = \frac{1}{x}$ to the graph of $y = \frac{2}{x}$, is by factor 2 and the point with coordinates $(1, 1)$ on the graph of $y = \frac{1}{x}$ is taken to the point with coordinates $(1, 2)$ on the graph of $y = \frac{2}{x}$.

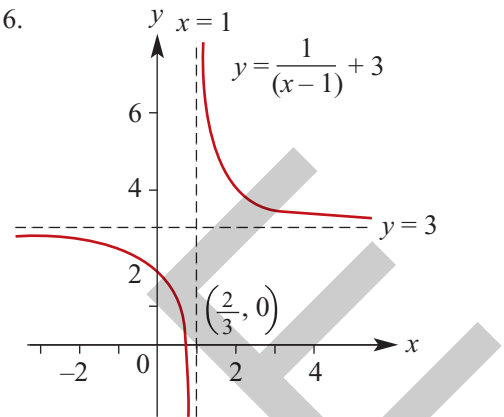


Dilations will be considered formally in the Chapter 6.

Now let us consider the graph of $y = \frac{1}{(x-1)} + 3$.

The basic graph of $y = \frac{1}{x}$ has been translated 1 unit to the right and 3 units up. The equation of the vertical asymptote is now $x = 1$ and the equation of the horizontal asymptote is now $y = 3$.

The graph now has x -axis and y -axis intercepts. These can be calculated in the usual way to give further detail to the graph.



x -axis intercept: let $y = 0$

y -axis intercept: let $x = 0$

$$0 = \frac{1}{x-1} + 3$$

$$y = \frac{1}{(0-1)} + 3$$

$$-3 = \frac{1}{x-1}$$

$$y = 2$$

$$-3(x-1) = 1$$

$$x = \frac{2}{3}$$

$$\therefore \text{intercept is } \left(\frac{2}{3}, 0\right)$$

$$\therefore \text{intercept is } (0, 2)$$

Using the above technique we are therefore able to sketch graphs of all rectangular hyperbolas of the form $y = \frac{a}{x-h} + k$.

Example 1

Sketch the graph of $y = \frac{2}{x+1} - 3$.

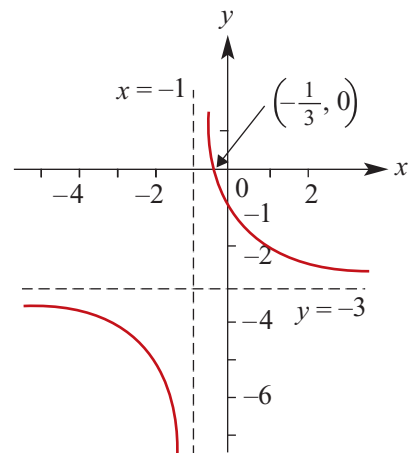
Solution

The graph of $\frac{2}{x}$ has been translated 1 unit to the left and 3 units down.

Asymptotes have equations $x = -1$ and $y = -3$.

y -axis intercept: $(0, -1)$

x -axis intercept: $\left(-\frac{1}{3}, 0\right)$



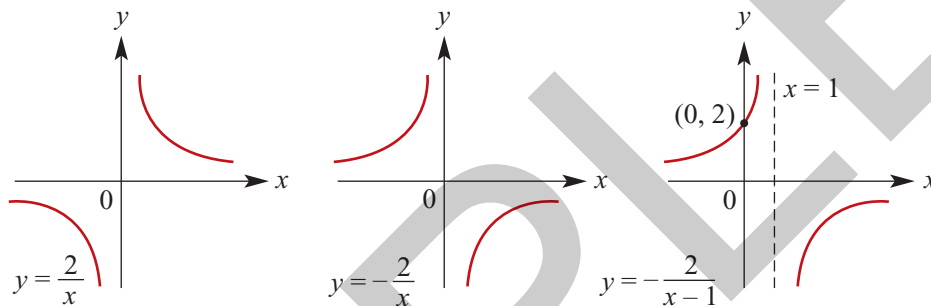
Example 2

Sketch the graph of $y = \frac{-2}{x-1}$.

Solution

The graph of $y = -\frac{2}{x}$ is obtained from the graph of $y = \frac{2}{x}$ by reflection in the x -axis.

This graph is then translated 1 unit to the right to obtain the graph of $y = \frac{-2}{x-1}$.

**Exercise 5A**

1 Sketch the graphs of the following, showing all important features of the graphs:

a $y = \frac{1}{x}$

b $y = \frac{2}{x}$

c $y = \frac{1}{2x}$

d $y = \frac{-3}{x}$

e $y = \frac{1}{x} + 2$

f $y = \frac{1}{x} - 3$

g $y = \frac{2}{x} - 4$

h $y = \frac{-1}{2x} + 5$

i $y = \frac{1}{x-1}$

j $y = \frac{-1}{x+2}$

k $y = \frac{1}{x+1} + 3$

l $y = \frac{-2}{x-3} - 4$

2 Write down the equations of the asymptotes for each of the graphs in Question 1.

5.2 The truncus

Consider the rule $y = \frac{1}{x^2} = x^{-2}$. Constructing a table of values for $y = \frac{1}{x^2}$ for values of x between -4 and 4 we have as follows:

x	-4	-3	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{9}$	$\frac{1}{4}$	1	4	4	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$

We can plot these points and then connect the dots to produce a continuous curve.

A graph of this shape is called a **truncus**.

Note that when $x = 0$, y is undefined, and that there is no x -value that will produce a negative value of y .

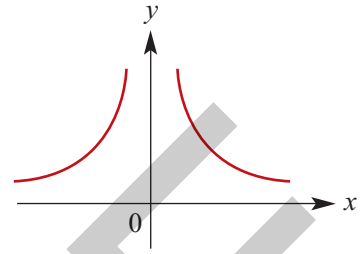
Note: As $x \rightarrow \infty$, $y \rightarrow 0^+$. As $x \rightarrow -\infty$, $y \rightarrow 0^+$.

As $x \rightarrow 0^+$, $y \rightarrow \infty$ and, as $x \rightarrow 0^-$, $y \rightarrow \infty$.

The graph of $y = \frac{1}{x^2}$, has asymptotes $y = 0$ and $x = 0$.

All graphs of the form $y = \frac{a}{(x-h)^2} + k$ will have the same basic 'truncus' shape.

Asymptotes will be the lines with equations $y = k$ and $x = h$.

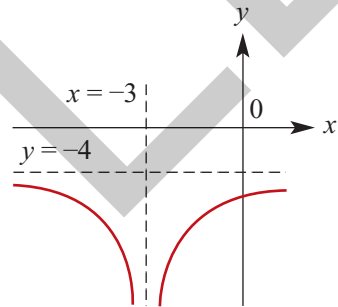


Example 3

Sketch the graph of $y = \frac{-1}{2(x+3)^2} - 4$.

Solution

The graph of $y = \frac{-1}{2x^2}$ is translated 3 units to the left and 4 units down.



Exercise 5B

1 Sketch the graphs of the following, showing all important features:

a $y = \frac{1}{(x+3)^2}$

b $y = \frac{1}{x^2} - 4$

c $y = \frac{-1}{(x-2)^2}$

d $y = \frac{1}{(x-1)^2} + 3$

e $y = \frac{1}{2(x+3)^2} - 4$

f $y = \frac{-2}{(x-2)^2} + 1$

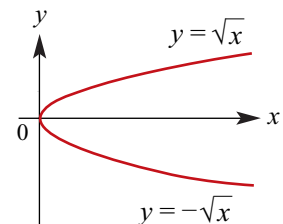
g $y = \frac{3}{(x+3)^2} - 6$

h $y = \frac{-1}{(x-4)^2} + 2$

2 Write down the equations of the asymptotes of each of the graphs in Question 1.

5.3 $y = \sqrt{x} = x^{1/2}$

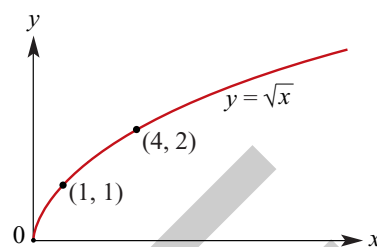
The rule $y = \sqrt{x} = x^{1/2}$ for $x \geq 0$ corresponds to the graph shown opposite. It is one arm of the parabola $x = y^2$.



Coordinates of points on the graph of $y = \sqrt{x}$ include $(0, 0)$, $(1, 1)$, $(4, 2)$ and $(9, 3)$.

All graphs of the form $y = a\sqrt{x-h} + k$ will have the same basic shape as the graph of $y = \sqrt{x}$.

The rule $y = \sqrt{-x}$ for $x \leq 0$ yields a graph which is the reflection of the graph of $y = \sqrt{x}$ in the y -axis. All graphs of the form $y = a\sqrt{-(x-h)} + k$ will have the same basic shape as the graph of $y = \sqrt{-x}$.



Example 4

Sketch the graph of $y = -2\sqrt{x-1} + 3$.

Solution

The graph is formed by dilating the graph of $y = \sqrt{x}$ by factor 2 from the x -axis, reflecting this in the x -axis and translating it 1 unit to the right and 3 units up.

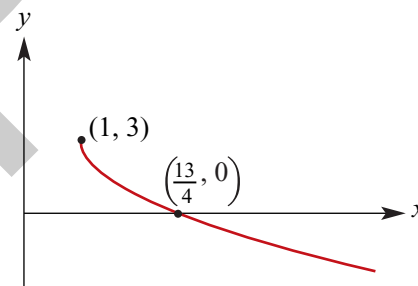
When $y = 0$, $-2\sqrt{x-1} + 3 = 0$.

Therefore $2\sqrt{x-1} = 3$.

Square both sides to give $4(x-1) = 9$

Therefore $x = \frac{9}{4} + 1 = \frac{13}{4}$.

The rule is defined for $x \geq 1$ and the values the rule can take (the range) is all numbers less than or equal to 3, i.e. $y \leq 3$.



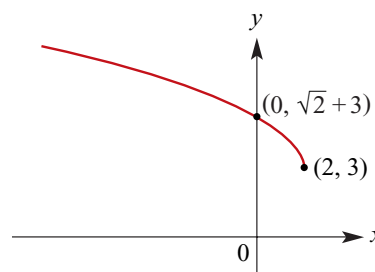
Example 5

Sketch the graph of $y = \sqrt{-(x-2)} + 3$.

Solution

When $x = 0$, $y = \sqrt{-(0-2)} + 3$
 $= \sqrt{2} + 3$

The rule is defined for $x \leq 2$ and the values the rule can take (the range) is all numbers greater than or equal to 3, i.e. $y \geq 3$.



Exercise 5C

For each of the following rules, sketch the corresponding graph, giving the axes intercepts when they exist, the set of x -values for which the rule is defined and the set of y -values which the rule takes:

- | | | |
|----------------------------------|------------------------------------|--------------------------------------|
| a $y = 2\sqrt{x} + 3$ | b $y = \sqrt{x - 2} + 3$ | c $y = \sqrt{x - 2} - 3$ |
| d $y = \sqrt{x + 2} + 1$ | e $y = -\sqrt{x + 2} + 3$ | f $y = 2\sqrt{x + 2} - 3$ |
| g $y = -\sqrt{x - 2} + 3$ | h $y = \sqrt{-(x - 4)} - 2$ | i $y = -2\sqrt{-(x + 4)} - 1$ |

5.4 Circles



The equation $x^2 + y^2 = r^2$ defines a circle whose centre is the origin $(0, 0)$, with radius r .

To the right is the graph of the circle with equation $x^2 + y^2 = 1$.

All circles can be considered as being transformations of this basic graph.

Changing the value of r obviously enlarges or reduces the circle.

As has been seen with other graphs, the basic graph may be translated horizontally and/or vertically.

The general equation for a circle is $(x - h)^2 + (y - k)^2 = r^2$.

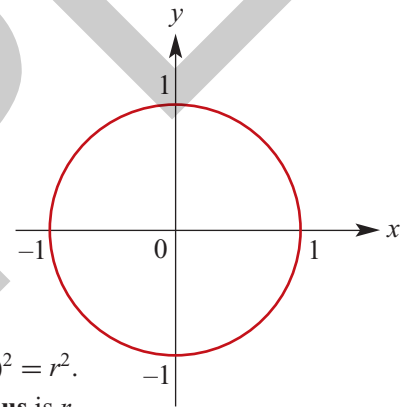
The **centre** of the circle is the point (h, k) and the **radius** is r .

If the radius and coordinates of the centre of the circle are given, the equation of the circle can be determined.

For example, if the radius is 2 and the centre is the point $(-3, 5)$ then the equation will be

$$\begin{aligned}(x - (-3))^2 + (y - 5)^2 &= 4 \\(x + 3)^2 + (y - 5)^2 &= 4\end{aligned}$$

If the equation of the circle is given, the radius and the centre of the circle can be determined and the graph sketched.



Example 6

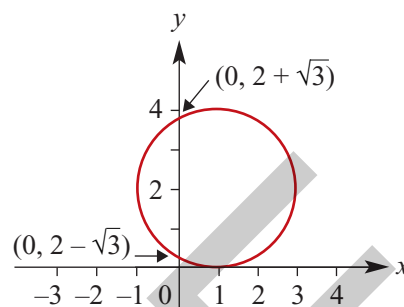
Sketch the graph of $(x - 1)^2 + (y - 2)^2 = 4$.

Solution

The equation $(x - 1)^2 + (y - 2)^2 = 4$ defines a circle of radius 2, with centre at the point $(1, 2)$.

When $x = 0$, $1 + (y - 2)^2 = 4$.

Therefore $(y - 2)^2 = 3$ and $y = 2 \pm \sqrt{3}$.

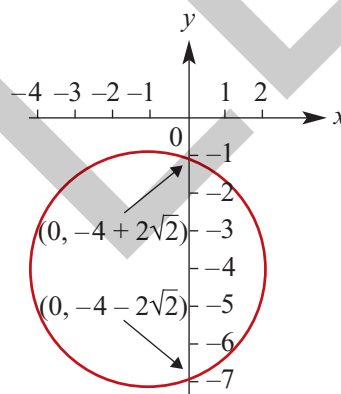
**Example 7**

Sketch the graph of the circle $(x + 1)^2 + (y + 4)^2 = 9$.

Solution

The circle has a radius of 3, and its centre at $(-1, -4)$. y -axis intercepts can be found in the usual way.

$$\begin{aligned} \text{When } x = 0, \quad 1 + (y + 4)^2 &= 9 \\ \text{i.e. } (y + 4)^2 &= 8 \\ y &= -4 \pm \sqrt{8} \\ &= -4 \pm 2\sqrt{2} \end{aligned}$$



The equation of a circle may not always be written in the form $(x - h)^2 + (y - k)^2 = r^2$.

Expanding the general equation of a circle gives

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ x^2 - 2hx + h^2 + y^2 - 2ky + k^2 &= r^2 \\ x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 &= 0 \end{aligned}$$

Let $h^2 + k^2 - r^2 = c$, then an alternative form of the circle equation is achieved:

$$x^2 + y^2 - 2hx - 2ky + c = 0$$

Notice that in the above form of the circle equation the coefficients of x^2 and y^2 are both 1 and there is no xy term.

In order to sketch a circle with equation expressed in this form, the equation can be converted to the ‘centre–radius’ form by completing the square for both x and y .

Example 8

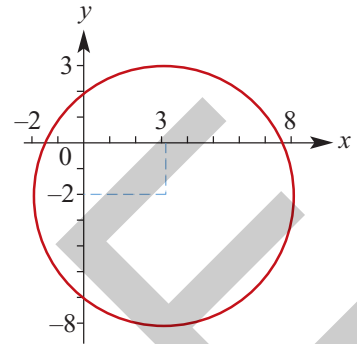
Find the radius and the coordinates of the centre of the circle with equation $x^2 + y^2 - 6x + 4y - 12 = 0$ and hence sketch the graph.

Solution

By completing the square for both x and y we have

$$\begin{aligned}x^2 + y^2 - 6x + 4y - 12 &= 0 \\(x^2 - 6x + 9) - 9 + (y^2 + 4y + 4) - 4 - 12 &= 0 \\(x^2 - 6x + 9) + (y^2 + 4y + 4) &= 25 \\(x - 3)^2 + (y + 2)^2 &= 5^2\end{aligned}$$

\therefore radius = 5, centre is at (3, -2).

**Semicircles**

Transposing the general equation of the circle $x^2 + y^2 = r^2$ to make y the subject, we have

$$\begin{aligned}y^2 &= r^2 - x^2 \\y &= \pm\sqrt{r^2 - x^2}\end{aligned}$$

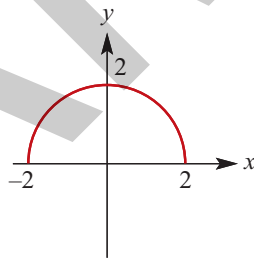
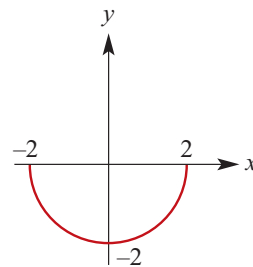
We can now consider two separate rules, $y = +\sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$, which represent the top half and bottom half of the circle respectively.

Example 9

Sketch the graphs of:

a $y = +\sqrt{4 - x^2}$

b $y = -\sqrt{4 - x^2}$

Solution**a****b****Exercise 5D**

Example 6 1 Write down the equations of the following circles with centres at $C(h, k)$ and radius r .

a $C(0, 0), r = 3$

b $C(0, 0), r = 4$

c $C(1, 3), r = 5$

d $C(2, -4), r = 3$

e $C(-3, 4), r = \frac{5}{2}$

f $C(-5, -6), r = 4.6$

2 Find the centre, C , and the radius, r , of the following circles:

Example 7

a $(x - 1)^2 + (y - 3)^2 = 4$

b $(x - 2)^2 + (y + 4)^2 = 5$

c $(x + 3)^2 + (y - 2)^2 = 9$

d $x^2 + y^2 - 6y - 16 = 0$

Example 8

e $(x + 3)^2 + (y + 2)^2 = 36$

f $x^2 + y^2 - 6x + 4y + 9 = 0$

g $x^2 + y^2 + 4x - 6y - 12 = 0$

h $x^2 + y^2 - 8x + 4y + 1 = 0$

3 Sketch the graphs of each of the following:

a $x^2 + y^2 = 64$

b $x^2 + (y - 4)^2 = 9$

c $(x + 2)^2 + y^2 = 25$

d $(x + 1)^2 + (y - 4)^2 - 169 = 0$

e $(2x - 3)^2 + (2y - 5)^2 = 36$

f $x^2 + y^2 - 6y - 16 = 0$

g $x^2 + y^2 + 4x - 6y - 3 = 0$

h $x^2 + y^2 - 8x + 22y + 27 = 0$

i $y = +\sqrt{9 - x^2}$

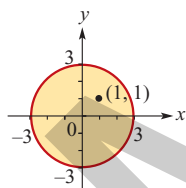
j $y = -\sqrt{25 - x^2}$

k $y = \sqrt{36 - (x - 2)^2}$

l $y - 2 = \sqrt{4 - (x + 2)^2}$

Example 9

- 4 Find the equation of the circle with centre $(2, -3)$ which touches the x -axis.
- 5 Find the equation of the circle whose centre is at the point $(2, 1)$ and which passes through the point $(4, -3)$.
- 6 Find the equation of the circle whose centre lies on the line $y = 4$ and which passes through the points $(2, 0)$ and $(6, 0)$.
- 7 The equation to a circle is $x^2 + y^2 + 4x - 6y = 23$. Find the centre and radius.
- 8 Find the length cut off on the x -axis and y -axis by the circle $x^2 + y^2 - 2x - 4y = 20$.
- 9 The graph of $x^2 + y^2 \leq 9$ is as shown.



Note that $(1, 1)$ satisfies $1^2 + 1^2 \leq 9$.

The coordinates of every point in the shaded region satisfy the inequality.

Sketch the graphs of each of the following. Use a dotted line to indicate that the boundary is not included.

a $x^2 + y^2 \leq 4$

b $x^2 + y^2 > 1$

c $x^2 + y^2 \leq 5$

d $x^2 + y^2 > 9$

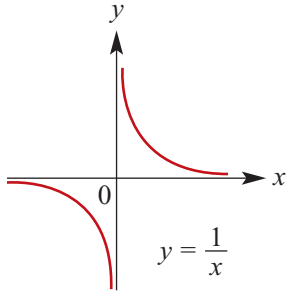
e $x^2 + y^2 \geq 6$

f $x^2 + y^2 < 8$

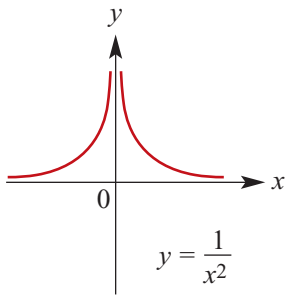


Chapter summary

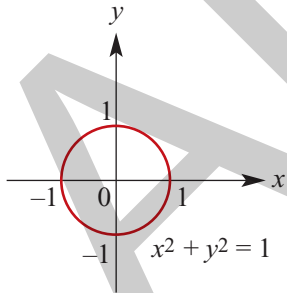
■ The standard graphs:



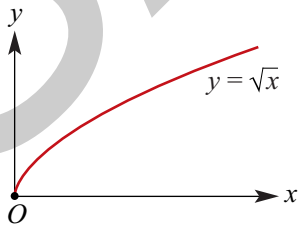
Rectangular hyperbola



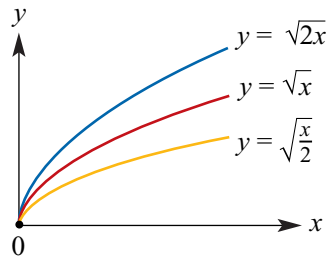
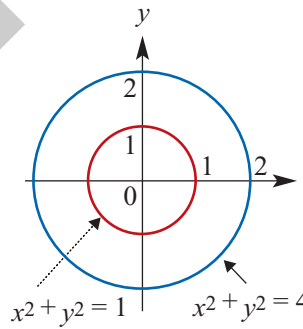
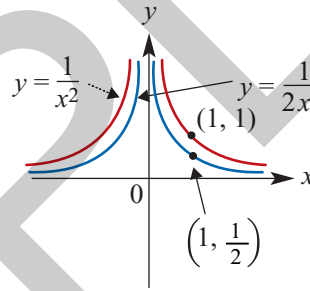
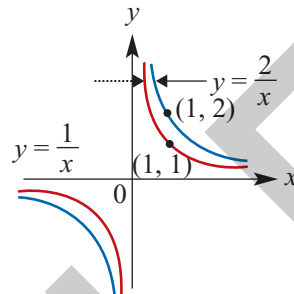
Truncus



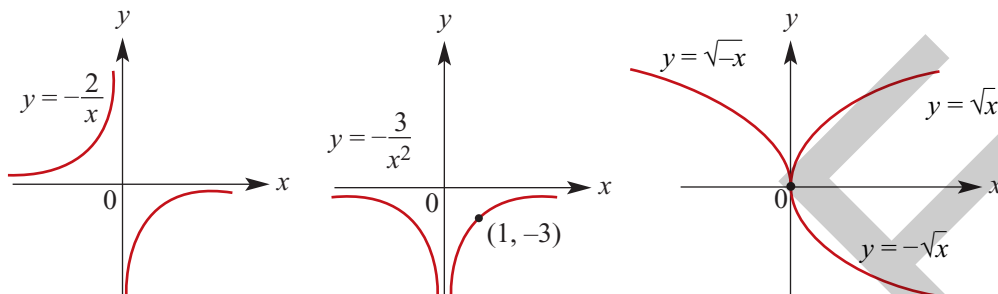
Circle



■ Dilations of these graphs:



■ Reflections in the axes:



■ General equation for a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

The centre is at (h, k) and the radius is r .

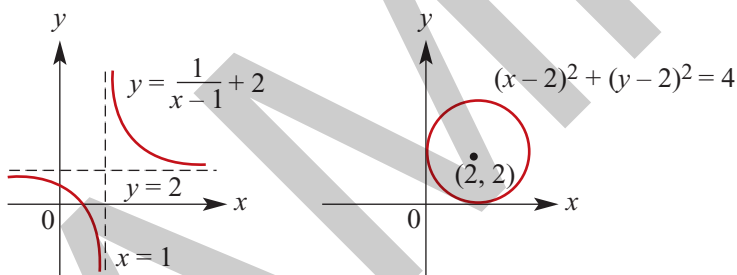
Alternative form:

$$x^2 + y^2 - 2hx - 2ky + c = 0$$

where $h^2 + k^2 - r^2 = c$.

■ Translations of graphs:

Examples



Multiple-choice questions

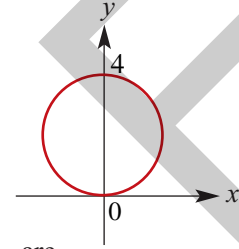
- The circle with equation $(x - a)^2 + (y - b)^2 = 36$ has its centre on the x -axis and passes through the point with coordinates $(6, 6)$. The values of a and b are

A $a = 0$ and $b = 6$	B $a = 0$ and $b = 0$	C $a = 2$ and $b = 0$
D $a = -6$ and $b = 0$	E $a = 6$ and $b = 0$	
- The equations of the asymptotes of the graph of $y = 5 - \frac{1}{3x - 5}$ are

A $x = 5, y = \frac{3}{5}$	B $y = 5, x = \frac{5}{3}$	C $x = 5, y = \frac{5}{3}$
D $y = 5, x = \frac{3}{5}$	E $x = 5, y = -\frac{5}{3}$	

- 3 For the rule $y = \frac{5}{x^2} + 3$, when $x = \frac{a}{2}$, $y =$
- A $\frac{5}{4a^2} + 3$ B $\frac{12a^2 + 5}{4a^2}$ C $\frac{20a^2 + 3}{4a^2}$
 D $\frac{12a^2 + 5}{a^2}$ E $\frac{20}{a^2} + 3$

- 4 If the y -axis is an axis of symmetry and the circle passes through the origin and $(0, 4)$, the equation of the circle shown is
- A $x^2 + (y - 2)^2 = 4$ B $(x - 2)^2 + y^2 = 2$
 C $(x + 2)^2 + y^2 = 4$ D $x^2 + (y - 2)^2 = 4$
 E $x^2 + y^2 - 2 = 4$



- 5 The equations of the asymptotes of the graph of $y = 5 + \frac{1}{(x - 2)^2}$ are
- A $x = 2, y = 5$ B $x = -2, y = 5$ C $x = 5, y = 4$
 D $x = 5, y = 2$ E $x = 4, y = 5$

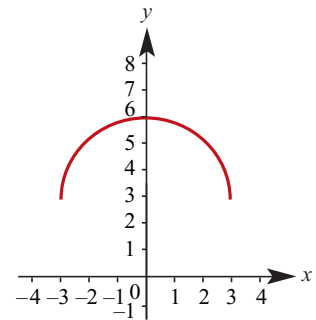
- 6 The coordinates of the centre and the radius of the circle with the equation $(x - 5)^2 + (y + 2)^2 = 9$ are
- A $(-5, 2)$ and 3 B $(-5, 2)$ and 9 C $(5, -2)$ and 9
 D $(5, -2)$ and 3 E $(-2, 5)$ and 3

- 7 For $y = -2\sqrt{x} + 3$ and $x \geq 0$, y will take the following values
- A $[3, \infty)$ B $(-3, \infty)$ C $[-3, \infty)$ D $(-\infty, 3]$ E $(3, \infty)$

- 8 The equation of the circle which has a diameter with end points at $(-2, 8)$ and $(6, 8)$ is
- A $(x + 2)^2 + (y - 8)^2 = 16$ B $(x + 2)^2 + (y + 8)^2 = 64$
 C $(x - 2)^2 + (y - 8)^2 = 16$ D $(x - 2)^2 + (y + 8)^2 = 4$
 E $(x - 2)^2 + (y - 8)^2 = 16$

- 9 Which of the following is the equation for the graph of a circle?
- A $y = 16 - x^2$ B $y^2 = 16 + x^2$ C $x + y^2 = 16$
 D $x^2 - y^2 = 16$ E $y^2 = 16 - x^2$

- 10 The equation of the semicircle shown is
- A $x^2 + (y - 3)^2 = 9$
 B $y = \sqrt{9 - x^2} + 3$
 C $y = \sqrt{9 + x^2} - 3$
 D $y = -\sqrt{9 - x^2} - 3$
 E $y = -\sqrt{9 - x^2} + 3$



Short-answer questions (technology-free)

1 Sketch the graphs of each of the following:

a $y = \frac{-3}{x}$

b $y = \frac{2}{x^2}$

c $y = \frac{1}{x-1}$

d $y = \frac{2}{x+1} + 1$

e $y = \frac{-2}{x^2}$

f $y = \frac{-1}{x-1}$

g $y = \frac{4}{2-x} + 3$

h $y = \frac{-3}{x^2} + 1$

i $y = 2\sqrt{x} + 2$

j $y = 2\sqrt{x-3} + 2$

k $y = -2\sqrt{x+2} + 2$

2 By completing the square, write each of the following in the form $(x-a)^2 + (y-b)^2 = r^2$:

a $x^2 + y^2 - 6x + 4y - 12 = 0$

b $x^2 + y^2 - 3x + 5y - 4 = 0$

c $2x^2 + 2y^2 - x + y - 4 = 0$

d $x^2 + y^2 + 4x - 6y = 0$

e $x^2 + y^2 = 6(x+y)$

f $x^2 + y^2 = 4x - 6y$

3 Find the equation of the diameter of the circle $x^2 + y^2 - 4x + 6y = 14$ which passes through the origin.

4 Find the equation of the diameter of the circle $x^2 + y^2 - 3x + 2y = 26$ which cuts the x -axis at an angle of 45° .

5 Find the equation to the circle with centre C and radius r for each of the following and sketch the graph:

a $C(3, 4), r = 5$

b $C(-1, 0), r = 1$

c $C(4, 4), r = 2$

d $C\left(\frac{1}{2}, \frac{-1}{3}\right), r = \frac{1}{6}$

6 Sketch the graphs of the following semicircles:

a $y = \sqrt{9-x^2}$

b $y = -\sqrt{16-(x+1)^2}$

c $y-2 = -\sqrt{1-x^2}$

d $y+3 = \sqrt{4-(x+2)^2}$

Extended-response questions

The following questions also involve techniques developed in chapters 2 and 4.

1 The line with equation $y = mx$ is tangent to the circle with centre $(10, 0)$ and radius 5 at the point $P(x, y)$.

a Find the equation of the circle.

b Show that the x -coordinate of the point P satisfies the equation $(1+m^2)x^2 - 20x + 75 = 0$.

c Use the discriminant for this equation to find the exact value of m .

d Find the coordinates of P . (There are two such points.)

e Find the distance of P from the origin.

- 2 A circle has its centre at the origin and radius 4.
- Find the equation of this circle.
 - Two lines which pass through the point $(8, 0)$ are tangents to this circle.
 - Show that the equations of these tangents are of the form $y = mx - 8m$.
 - Use techniques similar to those used in Question 1 to find the value of m and, hence, the equations of the tangents.
- 3 A circle has centre at the origin and radius 5. The point $P(3, 4)$ lies on the circle.
- Find the gradient of the line segment OP .
 - Find the gradient of the tangent to the circle at P .
 - Find the equation of the tangent at P .
 - If the tangent crosses the x -axis at A and the y -axis at B , find the length of line segment AB .
- 4 Let $P(x_1, y_1)$ be a point on the circle with equation $x^2 + y^2 = a^2$.
- Give the gradient of the line segment OP , where O is the origin.
 - Give the gradient of the tangent to the circle at P .
 - Show that the equation of the tangent at $P(x_1, y_1)$ is $x_1x + y_1y = a^2$.
 - If $x_1 = y_1$ and $a = 4$, find the equations of the possible tangents.
- 5 The line with equation $x = -a$ is the equation of the side BC of an equilateral triangle ABC circumscribing the circle with equation $x^2 + y^2 = a^2$.
- Find the equations of AB and AC .
 - Find the equation of the circle circumscribing triangle ABC .
- 6 Consider the curve with equation $y = \sqrt{x - b} + c$.
- Show that if the curve meets the line with equation $y = x$ at the point (a, a) then a satisfies the equation $a^2 - (2c + 1)a + c^2 + b = 0$.
 - If the line is a tangent to the curve, show that $c = \frac{4b - 1}{4}$.
 - Sketch the graph of $y = \sqrt{x} - \frac{1}{4}$ and find the coordinates of the point on the graph at which the line with equation $y = x$ is a tangent.
 - Find the values of k for which $y = x + k$:
 - meets the curve with equation $y = \sqrt{x} - \frac{1}{4}$ twice
 - meets the curve with equation $y = \sqrt{x} - \frac{1}{4}$ once
 - does not meet the curve with equation $y = \sqrt{x} - \frac{1}{4}$.
- 7 For the curve with equation $y = \sqrt{x} - 1$ and the straight line with equation $y = kx$, find the values of k such that:
- the line meets the curve twice
 - the line meets the curve once.