CAMBRIDGE TECHNOLOGY IN MATHS

Year 11
Quadratics for the TI-NSpire

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Example: Expanding and collecting like terms

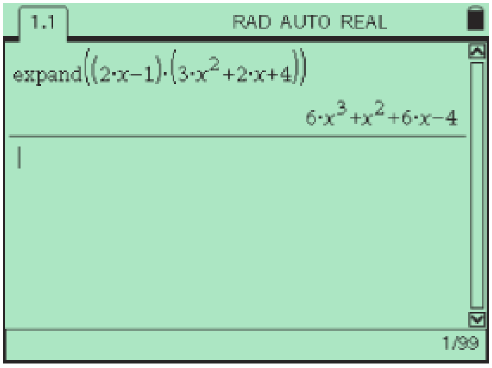
Expand $(2x - 1)(3x^2 + 2x + 4)$.

Solution

$$\begin{aligned}(2x-1)(3x^2+2x+4) &= 2x(3x^2+2x+4) - 1(3x^2+2x+4)\\ &= 6x^3+4x^2+8x-3x^2-2x-4\\ &= 6x^3+x^2+6x-4\end{aligned}$$

Using the TI-Nspire

Use **Expand( )** from the **Algebra** menu (menu 3 3) to expand the expression $(2x - 1)(3x^2 + 2x + 4)$.



Example: Factorising quadratics expressions

Factorise $6x^2 - 13x - 15$.

Solution

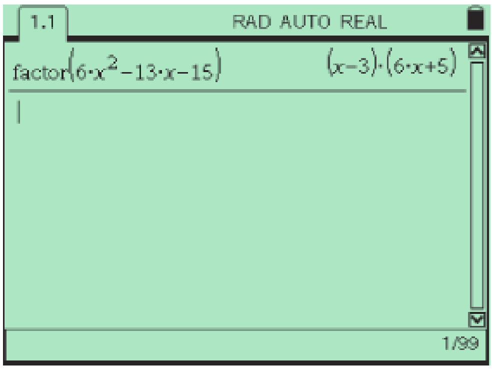
There are several combinations of factors of $6x^2$ and -15 to consider. Only one combination is correct.

Factors of $6x^2$	Factors of -15	'Cross-products' add to give $-13x$
$6x$	$+5$	$+5x$
x	-3	$-18x$
		$-13x$

$\therefore$ Factors of $6x^2 - 13x - 15$
$= (6x + 5)(x - 3)$

Using the TI-Nspire

Use **Factor( )** from the **Algebra** menu (menu 3 2) to factorise the expression $6x^2 - 13x - 15$.



Original location: Chapter 4 Example 16 (p.100)

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Example: Sketching quadratics in polynomial form

Find the x- and y-axis intercepts and the turning point, and hence sketch the graph of $y = x^2 + x - 12$.

Solution

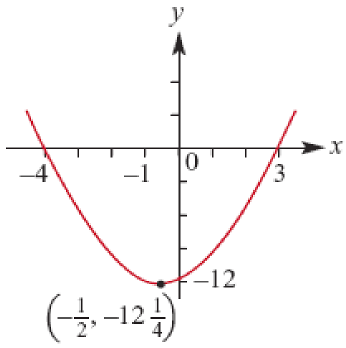
Step 1 $c = -12$, $\therefore$ y-axis intercept is $(0, -12)$

Step 2 Set $y = 0$ and factorise the right-hand side:

$$0 = x^2 + x - 12$$
$$0 = (x + 4)(x - 3)$$

$\therefore x = -4$ or $x = 3$

x-axis intercepts are $(-4, 0)$ and $(3, 0)$



Step 3 Due to the symmetry of the parabola, the axis of symmetry will be the line bisecting the two x-axis intercepts.

$\therefore$ the axis of symmetry is the line with equation $x = \dfrac{-4 + 3}{2} = -\dfrac{1}{2}$.

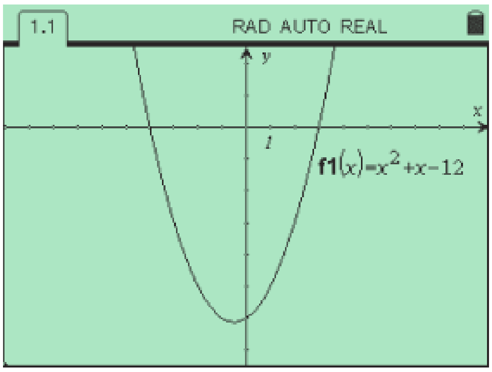
When $x = -\dfrac{1}{2}$, $y = \left(-\dfrac{1}{2}\right)^2 + \left(-\dfrac{1}{2}\right) - 12$

$$= -12\frac{1}{4}$$

$\therefore$ the turning point has coordinates $\left(-\dfrac{1}{2}, -12\dfrac{1}{4}\right)$.

Using the TI-Nspire

To graph the quadratic relation with rule $y = x^2 + x - 12$, enter the rule in the **Entry Line** of a **Graphs & Geometry** application as shown and press enter. Select the **Window Settings** (menu 4 1) $-10 \le x \le 10$ and $-15 \le y \le 5$ to obtain the graph as shown.



Example: The general quadratic formula

Solve each of the following equations for x by using the quadratic formula:

a $x^2 - x - 1 = 0$ **b** $x^2 - 2kx - 3 = 0$

Solution

a $x^2 - x - 1 = 0$

$a = 1, b = -1$ and $c = -1$

The formula gives

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

b $x^2 - 2kx - 3 = 0$

$a = 1, b = -2k$ and $c = -3$

The formula gives

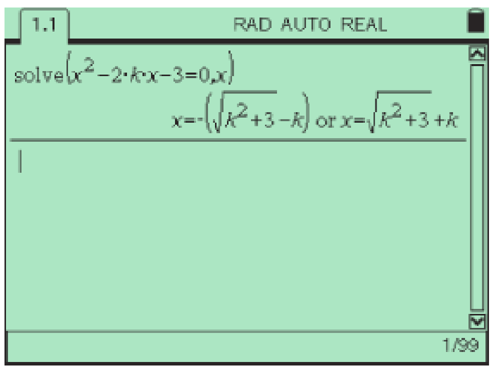
$$x = \frac{-(-2k) \pm \sqrt{(-2k)^2 - 4 \times 1 \times (-3)}}{2 \times 2}$$

$$= \frac{2k \pm \sqrt{4k^2 + 12}}{4}$$

$$= \frac{k \pm \sqrt{k^2 + 3}}{2}$$

Using the TI-Nspire

Use **Solve( )** from the **Algebra** menu (menu 3 1) to solve the equation $x^2 - 2kx - 3 = 0$ for x.



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Calculating iteration using the TI-Nspire

Using the TI-Nspire

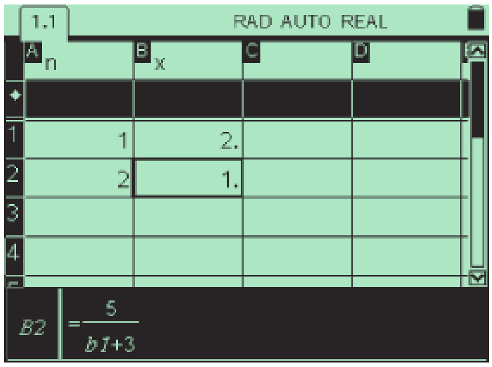
Method 1

Choose a starting value near the positive solution of $x^2 + 3x - 5 = 0$. Let $x_1 = 2$.

In a **Lists & Spreadsheet** application, enter 1 in cell A1 and 2. in cell B1.

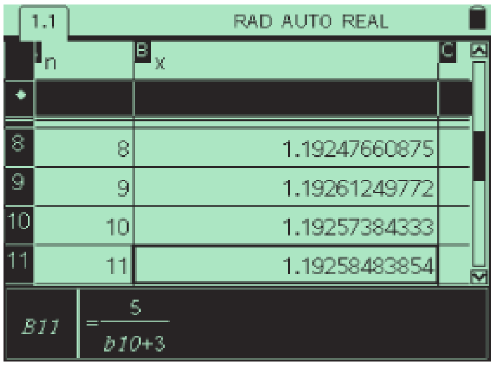
Enter = a1 + 1 in cell A2 and = 5/(b1 + 3) in cell B2.

(Entering 2. rather than 2 in B1 ensures the iterations are displayed as decimal numbers.)



Highlight the cells A2 and B2 using (shift) and the Nav Pad and use **Fill Down** (menu 3 3) to generate the sequence of iterations.

To better see the values of the iterations use **Maximize Column Width** (menu 1 2 2) for column B.

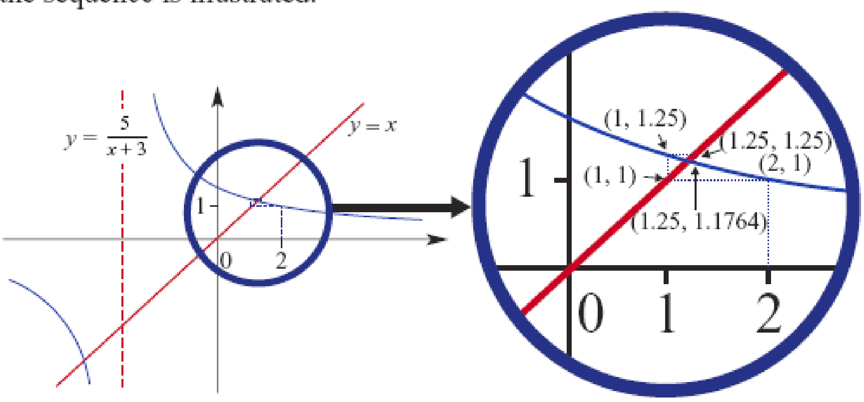


Try other starting values, for example $x_1 = -400$ or $x_1 = 200$.

Convergence is always towards the solution $x = \dfrac{-3 + \sqrt{29}}{2}$.

The convergence can be illustrated with a **cobweb** diagram.

The graphs of $y = \dfrac{5}{x + 3}$ and $y = x$ are sketched on the one set of axes and the 'path' of the sequence is illustrated.



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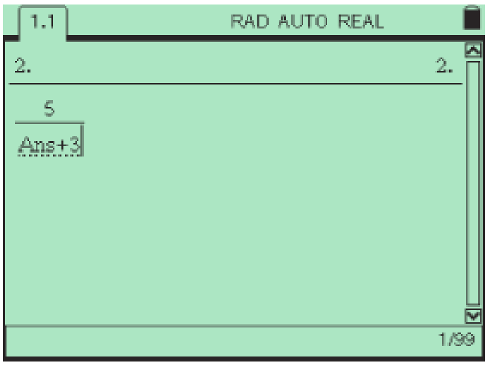
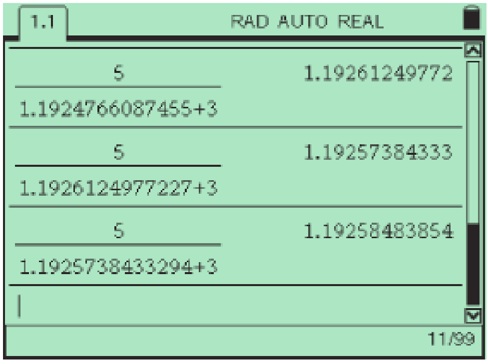
Method 2

Another method which can be used to generate the sequence is to use **ans** (ctrl (-)) in a **Calculator** application. To generate the sequence $x_n = \frac{5}{x_{n-1} + 3}$ first type 2 and press **enter**. Then type $\frac{5}{\text{ans} + 3}$ and press **enter** repeatedly to generate the sequence of iterations.





Example: Solving simultaneous linear and quadratic equations

Find the points of intersection of the line with the equation $y = -2x + 4$ and the parabola with the equation $y = x^2 - 8x + 12$.

Solution

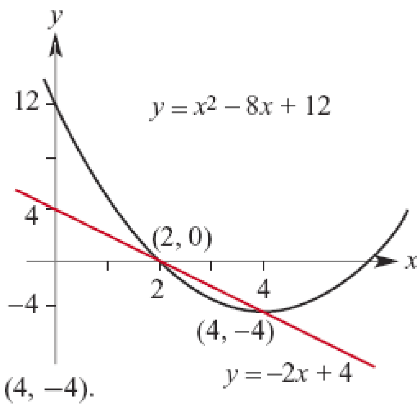
At the point of intersection

$$x^2 - 8x + 12 = -2x + 4$$
$$x^2 - 6x - 8 = 0$$
$$(x - 2)(x - 4) = 0$$

Hence $x = 2$ or $x = 4$

When $x = 2, y = -2(2) + 4 = 0$

$x = 4, y = -2(4) + 4 = -4$



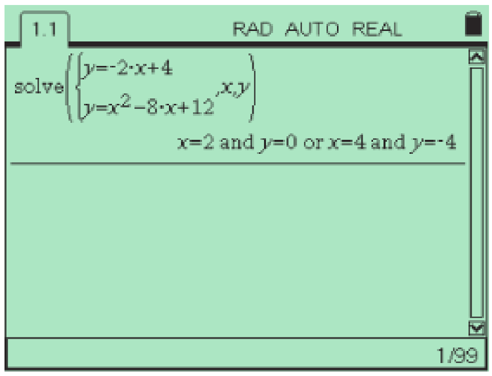
Therefore the points of intersection are (2, 0) and (4, −4).

The result can be shown graphically.

Using the TI-Nspire

Use **Solve( )** from the **Algebra** menu (menu 3 1) to solve the simultaneous equations $y = -2x + 4$ and $y = x^2 - 8x + 12$.

Access the simultaneous equations template (ctrl)

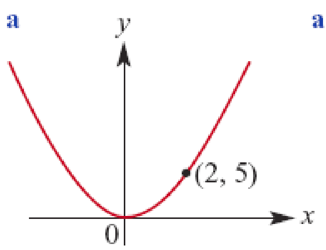


Example: Determining quadratic rules

Determine the quadratic rule for each of the following graphs, assuming each is a parabola.

Solution

a



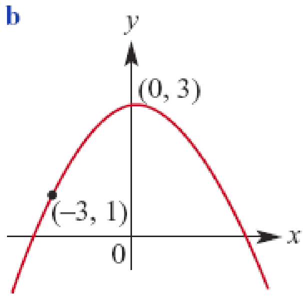
a This is of the form $y = ax^2$

When $x = 2, y = 5$, thus $5 = 4a$

Therefore $a = \frac{5}{4}$

and the rule is $y = \frac{5}{4}x^2$

b



b This is of the form $y = ax^2 + c$

For (0, 3) $3 = a(0) + c$

Therefore $c = 3$

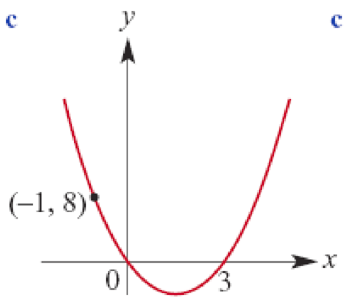
For (−3, 1) $1 = a(-3)^2 + 3$

$1 = 9a + 3$

Therefore $a = -\frac{2}{9}$

and the rule is $y = -\frac{2}{9}x^2 + 3$

c



c This is of the form $y = ax(x - 3)$

For (−1, 8) $8 = -a(-1 - 3)$

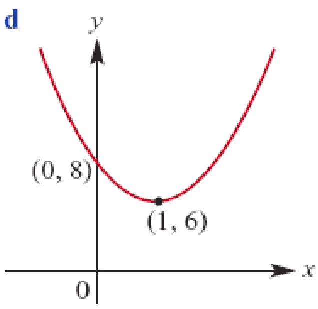
$8 = 4a$

Therefore $a = 2$

and the rule is $y = 2x(x - 3)$

$y = 2x^2 - 6x$

d



d This is of the form $y = k(x - 1)^2 + 6$

When $x = 0$, $y = 8$

$\therefore$ $8 = k + 6$

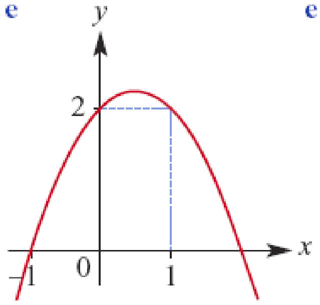
and $k = 2$

$\therefore$ $y = 2(x - 1)^2 + 6$

and the rule is $y = 2(x^2 - 2x + 1) + 6$

$y = 2x^2 - 4x + 8$

e



e This is of the form $y = ax^2 + bx + c$

For $(-1, 0)$ $\quad 0 = a - b + c \quad (1)$

For $(0, 2)$ $\quad 2 = c \quad (2)$

For $(1, 2)$ $\quad 2 = a + b + c \quad (3)$

Substitute $c = 2$ in (1) and (3)

$$0 = a - b + 2$$

$$-2 = a - b \quad (1a)$$

$$0 = a + b \quad (3a)$$

Substract (3a) from (1a)

$$-2 = -2b$$

Therefore $\quad b = 1$

Substitute $b = 1$ and $c = 2$ in (1)

Therefore $\quad 0 = a - 1 + 2$

$$0 = a + 1$$

and hence $\quad a = -1$

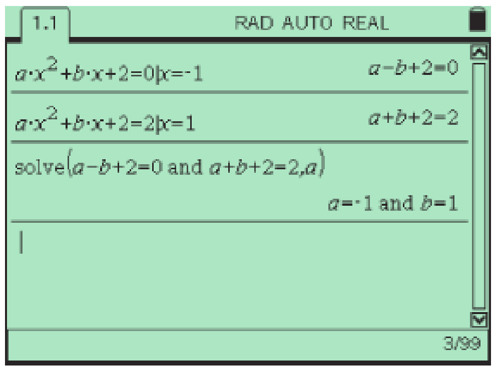
Thus the quadratic rule is $\quad y = -x^2 + x + 2$

Using the TI-Nspire

The equation $y = ax^2 + bx + 2$ and the two points $(-1, 0)$ and $(1, 2)$ are used to generate equations in a and b.

These equations are then solved simultaneously to find a and b.

Note: Here we have used **and**. The simultaneous equations template can also be used.



Original location: Chapter 4 Example 37 (p.130-132)

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