CAMBRIDGE TECHNOLOGY IN MATHS

Year 11
Quadratics for the TI-89

CONTENTS

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Example: Expanding and collecting like terms

Expand $(2x - 1)(3x^2 + 2x + 4)$.

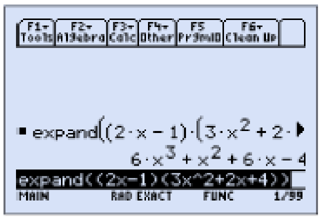
Solution

$$\begin{aligned}(2x-1)(3x^2+2x+4) &= 2x(3x^2+2x+4) - 1(3x^2+2x+4)\\ &= 6x^3 + 4x^2 + 8x - 3x^2 - 2x - 4\\ &= 6x^3 + x^2 + 6x - 4\end{aligned}$$

Using a CAS calculator

Press F2 and from the **Algebra** menu select **3:expand** to expand the expression $(2x-1)\left(3x^2+2x+4\right)$.

Complete as shown in the entry line.



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Example: Factorising quadratics expressions

Factorise $6x^2 - 13x - 15$.

Solution

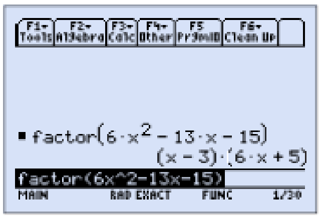
There are several combinations of factors of $6x^2$ and -15 to consider. Only one combination is correct.

Factors of $6x^2$	Factors of -15	'Cross-products' add to give $-13x$
$6x$	$+5$	$+5x$
x	-3	$-18x$
		$-13x$

$\therefore$ Factors of $6x^2 - 13x - 15$
$= (6x + 5)(x - 3)$

Using a CAS calculator

Press F2 and from the **Algebra** menu select **2:factor** to factorise the expression $6x^2 - 13x - 15$.



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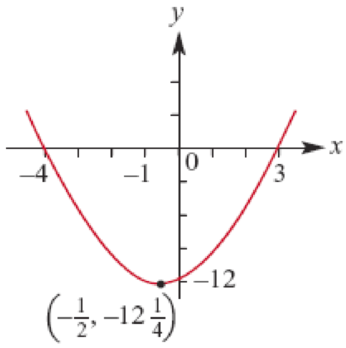
Example: Sketching quadratics in polynomial form

Find the x- and y-axis intercepts and the turning point, and hence sketch the graph of $y = x^2 + x - 12$.

Solution



Step 1 $c = -12$, $\therefore$ y-axis intercept is $(0, -12)$

Step 2 Set $y = 0$ and factorise the right-hand side:

$$0 = x^2 + x - 12$$
$$0 = (x + 4)(x - 3)$$

$\therefore x = -4$ or $x = 3$

x-axis intercepts are $(-4, 0)$ and $(3, 0)$

Step 3 Due to the symmetry of the parabola, the axis of symmetry will be the line bisecting the two x-axis intercepts.

$\therefore$ the axis of symmetry is the line with equation $x = \dfrac{-4 + 3}{2} = -\dfrac{1}{2}$.

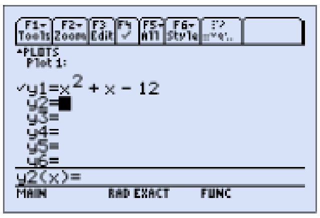
When $x = -\dfrac{1}{2}$, $y = \left(-\dfrac{1}{2}\right)^2 + \left(-\dfrac{1}{2}\right) - 12$

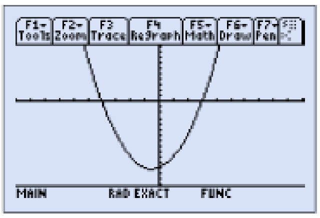
$$= -12\frac{1}{4}$$

$\therefore$ the turning point has coordinates $\left(-\dfrac{1}{2}, -12\dfrac{1}{4}\right)$.

Using a CAS calculator

To graph the quadratic relation with rule $y = x^2 + x - 12$, enter the rule in the **Y=** screen as shown. Press [♦] [GRAPH] to obtain the graph.





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Example: The general quadratic formula

Solve each of the following equations for x by using the quadratic formula:

a $x^2 - x - 1 = 0$ **b** $x^2 - 2kx - 3 = 0$

Solution

a $x^2 - x - 1 = 0$

$a = 1$, $b = -1$ and $c = -1$

The formula gives

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

b $x^2 - 2kx - 3 = 0$

$a = 1$, $b = -2k$ and $c = -3$

The formula gives

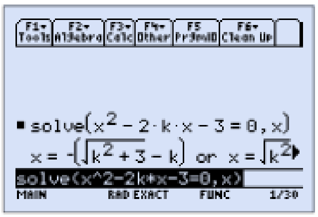
$$x = \frac{-(-2k) \pm \sqrt{(-2k)^2 - 4 \times 1 \times (-3)}}{2 \times 2}$$

$$= \frac{2k \pm \sqrt{4k^2 + 12}}{4}$$

$$= \frac{k \pm \sqrt{k^2 + 3}}{2}$$

Using a CAS calculator

Use **1:solve** to solve the equation $x^2 - 2kx - 3 = 0$ for x.



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Calculating iteration using the TI-89

Using a CAS calculator

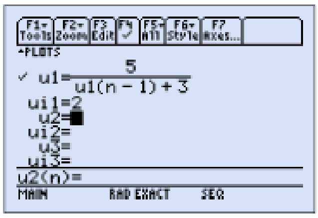
Choose a starting value near the positive solution of $x^2 + 3x - 5 = 0$. Let $x_1 = 2$.

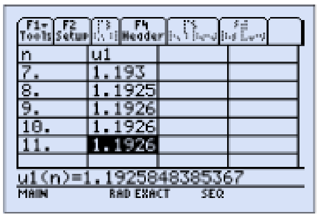
Enter this iterative equation in the CAS calculator as shown. Note that **4:Sequence** has been chosen from the **1:Function** submenu of the **MODE** menu.

Press [♦] and [TABLE] to obtain a table of values.

$x_1 = 2$, $x_2 = 1$, $x_3 = 1.25$, $x_4 = 1.1764...$, $x_{15} = 1.19258241...$, $x_{16} =$ 1.19258239...

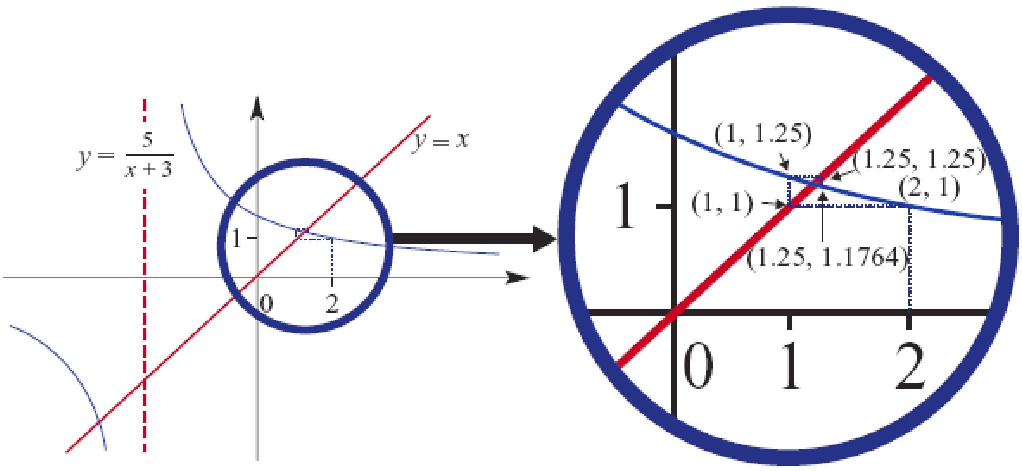
The sequence converges to $\dfrac{-3+\sqrt{29}}{2}$.





The convergence can be illustrated with a **web** diagram.

The graphs of $y = \dfrac{5}{x+3}$ and $y = x$ are sketched on the one set of axes and the 'path' of the sequence is illustrated.



Try other starting points, for example $x_1 = -400$ or $x_1 = 200$.

Convergence is always towards the solution $x = \dfrac{-3+\sqrt{29}}{2}$.

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Other arrangements of the equation $x^2 + 3x - 5 = 0$ can be tried to find the other solution. For example:

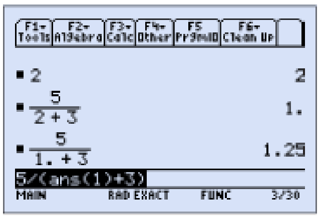
A $x = \dfrac{-x^2 + 5}{3}$ yields the sequence $x_{n+1} = \dfrac{-(x_n)^2 + 5}{3}$. This converges to $\dfrac{-3 + \sqrt{29}}{2}$ again if $x_1 = 2$ is used for the start. $x_1 = -9$ yields no convergence.

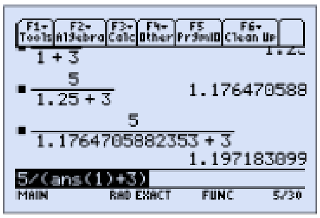
B $x = \dfrac{5}{x} - 3$ yields the sequence $x_{n+1} = \dfrac{5}{x_n} - 3$. This converges to $\dfrac{-3 - \sqrt{29}}{2}$ for the start $x_1 = -9$.

C $x = -\sqrt{5 - 3x}$ yields the sequence $x_{n+1} = -\sqrt{5 - 3x_n}$. This converges to $\dfrac{-3 - \sqrt{29}}{2}$ for the start $x_1 = -10$.

Another method which can be used to generate the sequence is to use **ANS** in the Home screen. Thus for $x_{n+1} = \dfrac{5}{x_n + 3}$ first enter 2 and press ENTER and then complete as $5/(\text{ans}\,(1) + 3)$.

Press ENTER repeatedly to generate the sequence.





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Example: Solving simultaneous linear and quadratic equations

Find the points of intersection of the line with the equation $y = -2x + 4$ and the parabola with the equation $y = x^2 - 8x + 12$.

Solution

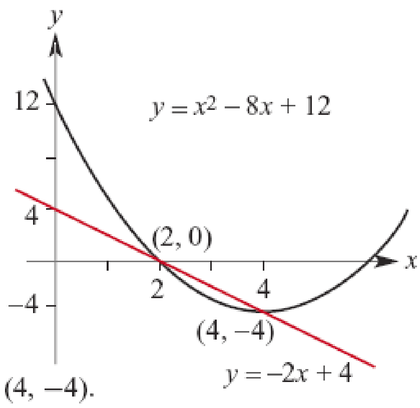
At the point of intersection

$$x^2 - 8x + 12 = -2x + 4$$
$$x^2 - 6x - 8 = 0$$
$$(x - 2)(x - 4) = 0$$

Hence $x = 2$ or $x = 4$

When $x = 2, y = -2(2) + 4 = 0$

$x = 4, y = -2(4) + 4 = -4$

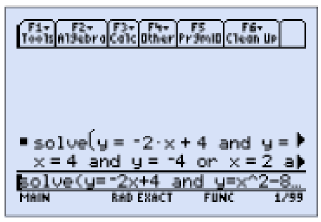


Therefore the points of intersection are (2, 0) and (4, −4).

The result can be shown graphically.

Using a CAS calculator

Use **1:solve** to solve the simultaneous equations $y = -2x + 4$ and $y = x^2 - 8x + 12$.



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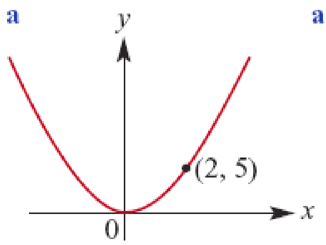
Example: Determining quadratic rules

Determine the quadratic rule for each of the following graphs, assuming each is a parabola.

Solution

a



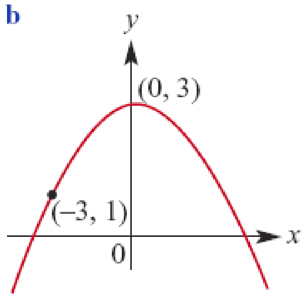
a This is of the form $y = ax^2$

When $x = 2, y = 5$, thus $5 = 4a$

Therefore $a = \frac{5}{4}$

and the rule is $y = \frac{5}{4}x^2$

b



b This is of the form $y = ax^2 + c$

For (0, 3) $3 = a(0) + c$

Therefore $c = 3$

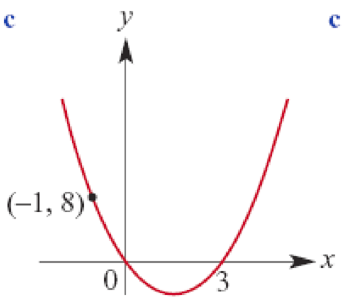
For (−3, 1) $1 = a(-3)^2 + 3$

$1 = 9a + 3$

Therefore $a = -\frac{2}{9}$

and the rule is $y = -\frac{2}{9}x^2 + 3$

c



c This is of the form $y = ax(x - 3)$

For (−1, 8) $8 = -a(-1 - 3)$

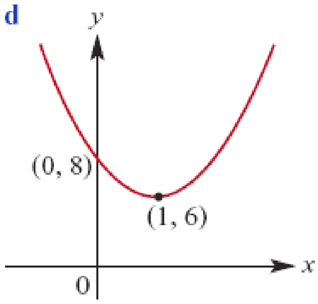
$8 = 4a$

Therefore $a = 2$

and the rule is $y = 2x(x - 3)$

$y = 2x^2 - 6x$

d



d This is of the form $y = k(x - 1)^2 + 6$

When $x = 0$, $y = 8$

$\therefore$ $8 = k + 6$

and $k = 2$

$\therefore$ $y = 2(x - 1)^2 + 6$

and the rule is $y = 2(x^2 - 2x + 1) + 6$

$y = 2x^2 - 4x + 8$

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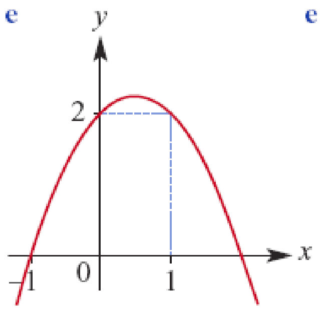


This is of the form $y = ax^2 + bx + c$

For $(-1, 0)$ $\quad 0 = a - b + c \quad (1)$

For $(0, 2)$ $\quad 2 = c \quad (2)$

For $(1, 2)$ $\quad 2 = a + b + c \quad (3)$

Substitute $c = 2$ in (1) and (3)

$$0 = a - b + 2$$
$$-2 = a - b \quad (1a)$$
$$0 = a + b \quad (3a)$$

Substract (3a) from (1a)

$$-2 = -2b$$

Therefore $\quad b = 1$

Substitute $b = 1$ and $c = 2$ in (1)

Therefore $\quad 0 = a - 1 + 2$

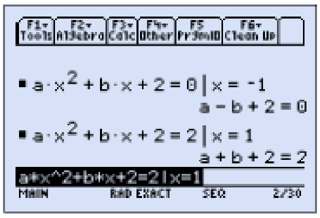
$$0 = a + 1$$

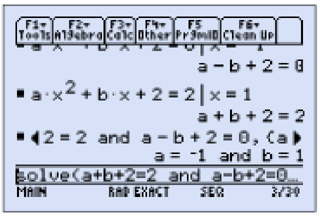
and hence $\quad a = -1$

Thus the quadratic rule is $\quad y = -x^2 + x + 2$

Using a CAS calculator

The equation $y = ax^2 + bx + 2$ is used to generate equations in a and b. These equations are then solved simultaneously to find a and b.





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