CAMBRIDGE TECHNOLOGY IN MATHS

Year 12
Functions and relations for the ClassPad

CONTENTS

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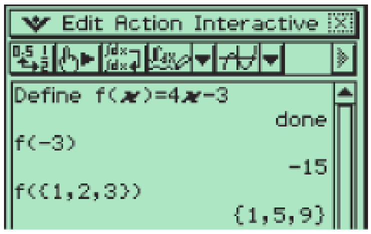
Function notation using the ClassPad

Using the Casio ClassPad

Function notation can be used with a CAS calculator.
In Main tap **Interactive–Define** and enter the function name, variable and expression as shown.
See page 10 for a screen showing the **Define** window.
Enter $f(-3)$ in the entry line and tap EXE.
In the entry line, type $f(\{1,2,3\})$ to obtain the values of $f(1)$, $f(2)$ and $f(3)$.



Examples: Function notation

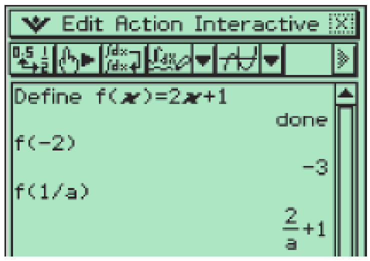
If $f(x) = 2x + 1$, find $f(-2)$ and $f\left(\frac{1}{a}\right)$, $a \neq 0$.

Solution

$$f(-2) = 2(-2) + 1 = -3$$
$$f\left(\frac{1}{a}\right) = 2\left(\frac{1}{a}\right) + 1 = \frac{2}{a} + 1$$

Using the Casio ClassPad

In Main tap **Interactive–Define** and enter the function name f, variable x and expression $2x + 1$.
Now complete $f(-2)$ and $f\left(\frac{1}{a}\right)$.



Original location: Chapter 1 Example 5-6 (p.8-10)

Consider the function defined by $f(x) = 2x - 4$ for all $x \in R$.

a Find the value of $f(2)$, $f(-1)$ and $f(t)$.

b For what values of t is $f(t) = t$?

c For what values of x is $f(x) \geq x$?

d Find the pre-image of 6.

Solution

a
$$\begin{aligned} f(2) &= 2(2) - 4 \\ &= 0 \\ f(-1) &= 2(-1) - 4 \\ &= -6 \\ f(t) &= 2t - 4 \end{aligned}$$

b
$$\begin{aligned} f(t) &= t \\ 2t - 4 &= t \\ t - 4 &= 0 \\ \therefore t &= 4 \end{aligned}$$

c
$$\begin{aligned} f(x) &\geq x \\ 2x - 4 &\geq x \\ x - 4 &\geq 0 \\ \therefore x &\geq 4 \end{aligned}$$

d
$$\begin{aligned} f(x) &= 6 \\ 2x - 4 &= 6 \\ x &= 5 \end{aligned}$$

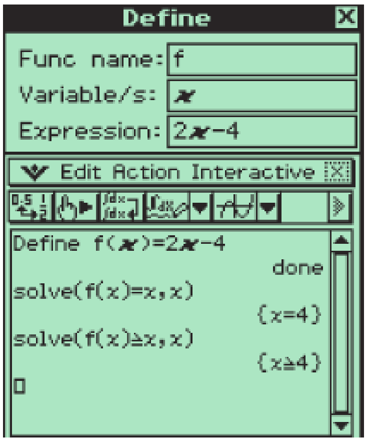
5 is the pre-image of 6.

Using the Casio ClassPad

Tap **Interactive–Define** and enter the function name, variable and expression as shown.

Enter and highlight $f(x) = x$, tap **Interactive–Equation/inequality–solve** and ensure the variable is set as x.

To enter the inequality, press Keyboard and look in the mth OPTN to find the $\geq$ symbol.



Example: Graphing functions

Sketch the graphs of each of the following functions and state the range of each of the functions:

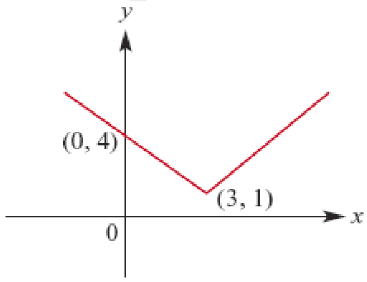
a $f(x) = |x - 3| + 1$ **b** $f(x) = -|x - 3| + 1$

Solution

First, note that $|a - b| = a - b$ if $a \geq b$ and $|a - b| = b - a$ if $b \geq a$.

a $f(x) = |x - 3| + 1 = \begin{cases} x - 3 + 1 & \text{if } x \geq 3 \\ 3 - x + 1 & \text{if } x < 3 \end{cases}$

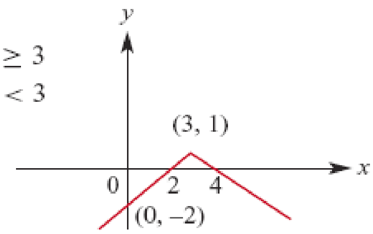
$= \begin{cases} x - 2 & \text{if } x \geq 3 \\ 4 - x & \text{if } x < 3 \end{cases}$



Range $= [1, \infty)$

b $f(x) = -|x - 3| + 1 = \begin{cases} -(x - 3) + 1 & \text{if } x \geq 3 \\ -(3 - x) + 1 & \text{if } x < 3 \end{cases}$

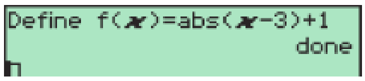
$= \begin{cases} -x + 4 & \text{if } x \geq 3 \\ -2 + x & \text{if } x < 3 \end{cases}$



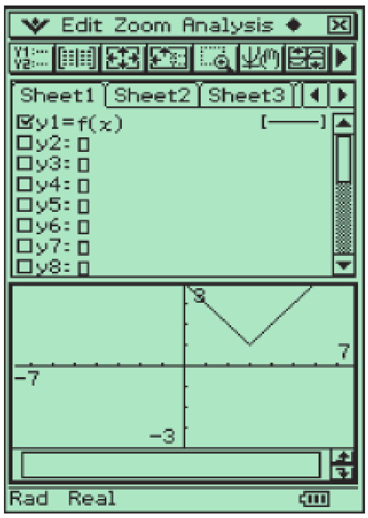
Range $= (-\infty, 1]$

Using the Casio ClassPad

Tap **Interactive-Define** and enter the function name, variable and function as shown.



To enter the absolute value, press Keyboard and look in the mth OPTN to find the |x| symbol.



In Graph&Tab enter $f(x)$ into $y1$, tick the box to select and tap to create the graph.

Note that the expression could be directly entered in the '$y1 =$' line but this gives you greater flexibility to use the function in other ways if required.

Example: Composite functions

Find both $f \circ g$ and $g \circ f$, stating the domain and range of each where:

$$f: R \to R,\ f(x) = 2x - 1 \quad \text{and} \quad g: R \to R,\ g(x) = 3x^2$$

Solution

To determine the existence of a composite function, it is useful to form a table of domains and ranges.

	Domain	Range
g	R	$R^+ \cup \{0\}$
f	R	R

$f \circ g$ is defined since ran $g \subseteq$ dom f, and $g \circ f$ is defined since ran $f \subseteq$ dom g.

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f(3x^2) \\ &= 2(3x^2) - 1 \\ &= 6x^2 - 1 \end{aligned}$$

and dom $f \circ g =$ dom $g = R$ and ran $f \circ g = [-1, \infty)$

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(2x - 1) \\ &= 3(2x - 1)^2 \\ &= 12x^2 - 12x + 3 \end{aligned}$$

$$\begin{aligned} \text{dom } g \circ f &= \text{dom } f \\ &= R \\ \text{ran } g \circ f &= [0, \infty) \end{aligned}$$

It can be seen from this example that in general $f \circ g \neq g \circ f$.

Using the Casio ClassPad

Define $f(x) = 2x - 1$ and $g(x) = 3x^2$. The rules for $f \circ g$ and $g \circ f$ can now be found using $f(g(x))$ and $g(f(x))$.